

Pressure vessels

Open end

a) thin wall b) thick wall

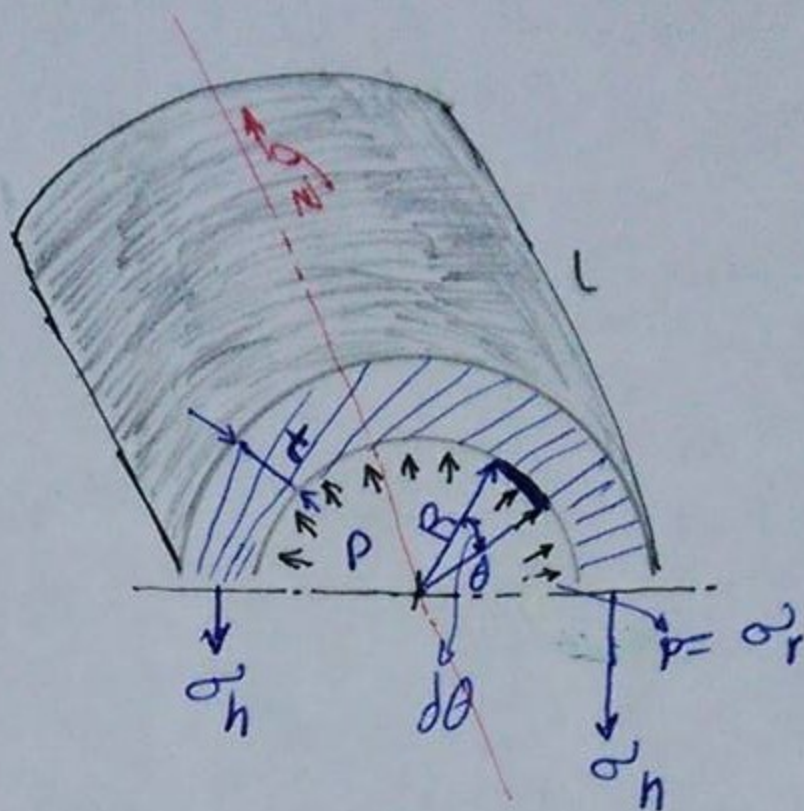
2) closed end Tanks

For thin wall pipes:

$$\frac{t}{D} < 0.1$$

σ_h = hoop stress

σ_r = radial stress



$$\Sigma \text{ Force} = 0$$

$$2\sigma_h \cdot t \cdot L = \int_0^\pi P \cdot R d\theta \cdot L \cdot \sin\theta$$

$$\sigma_h = \frac{PR}{2t} [-\cos\theta]_0^\pi$$

$$\sigma_h = \frac{PR}{2t}$$

$$\sigma_r = P \quad , \quad \sigma_h = \frac{PR}{2t}$$

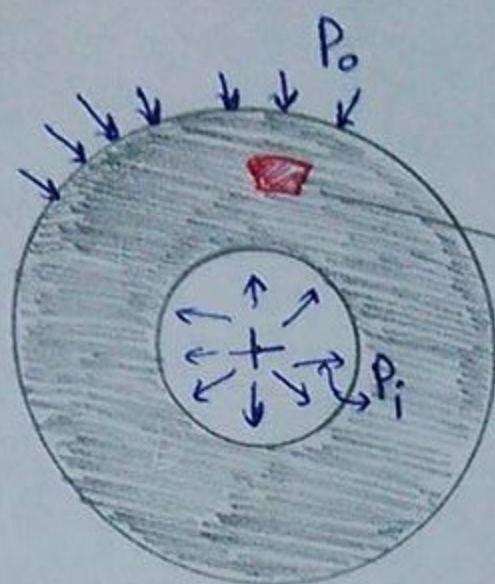
For open end $\sigma_z = 0$

For closed end

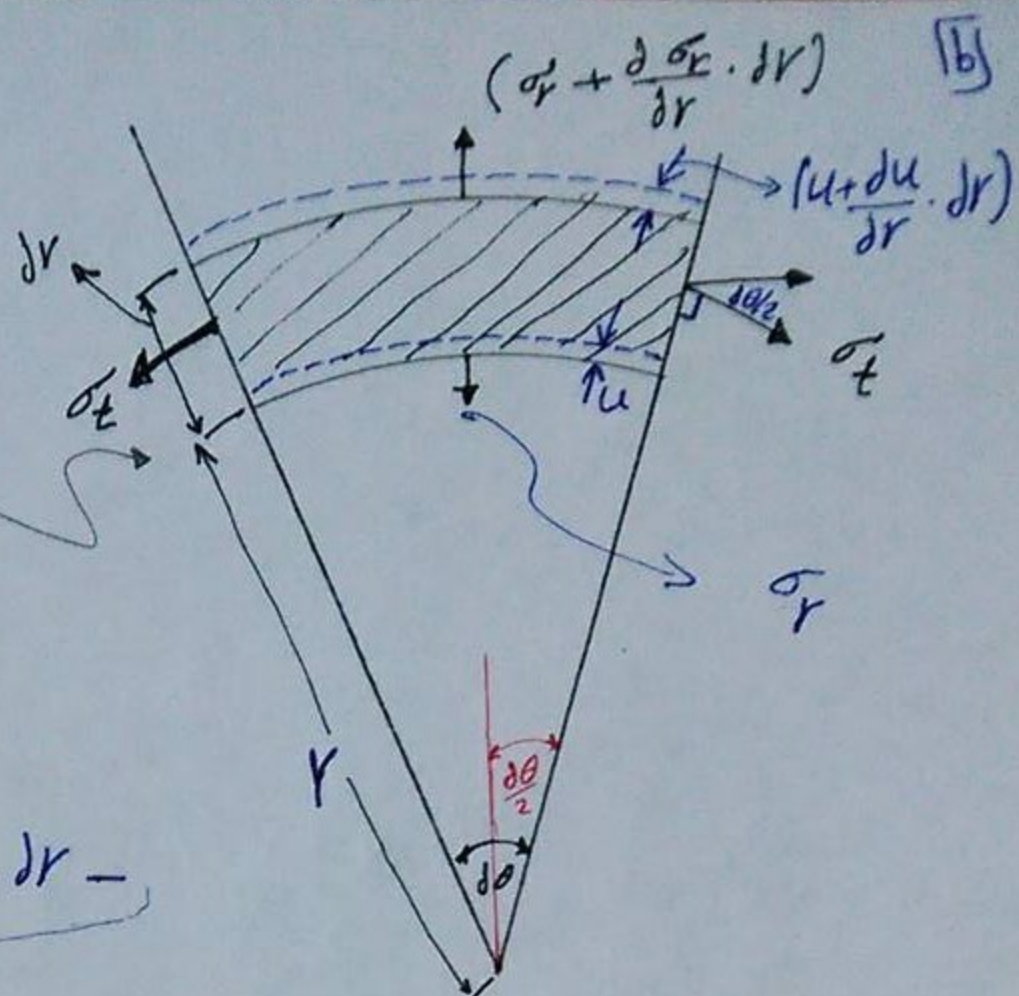
$$2\pi R t \cdot \sigma_z = P \cdot \frac{\pi D^2}{4}$$

$$\sigma_z = \frac{PD}{4t} = \frac{\sigma_h}{2}$$

b- for thick wall



بأخذ شريحة



$$\sigma_r \cdot r d\theta \cdot \frac{1}{2} + 2 \frac{\sigma_t}{2} \sin \frac{d\theta}{2} \cdot dr -$$

← مساحة الشريحة

$$\rightarrow (\sigma_r + \frac{d\sigma_r}{dr} \cdot dr) (r+dr) \cdot d\theta$$

$$\sigma_r \cdot r d\theta + \sigma_t \cdot dr \cdot d\theta - \sigma_r \cdot r \cdot d\theta - \sigma_r \cdot dr d\theta - r \frac{d\sigma_r}{dr} \cdot dr \cdot d\theta - \frac{d\sigma_r}{dr} \cdot (dr)^2 \cdot d\theta = 0$$

$$\sigma_t - \sigma_r - r \frac{d\sigma_r}{dr} = 0 \rightarrow (1)$$

$$\epsilon_r = \frac{\partial u}{\partial r} \rightarrow (2)$$

$$\epsilon_t = \frac{u}{r} \rightarrow (3)$$

$$\sigma_r = \frac{E}{(1-n)} \left(\frac{\partial u}{\partial r} + n \frac{u}{r} \right) \rightarrow (4)$$

$$\sigma_t = \frac{E}{1-n^2} \left(\frac{u}{r} + n \frac{\partial u}{\partial r} \right) \rightarrow (5)$$

بالتعويض من (4) في (1)

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \cdot \frac{du}{dr} - \frac{u}{r^2} = 0 \rightarrow (6)$$

$$u = C_1 r + \frac{C_2}{r} \rightarrow (7)$$

$$\sigma_r = A - \frac{B}{r^2} \rightarrow (8)$$

$$\sigma_t = A + \frac{B}{r^2} \rightarrow (9)$$

Stresses produced by internal or external pressures.

Case (a) Internal pressure only.

at $r=a$

$$\sigma_r = -P_i$$

$r=b$

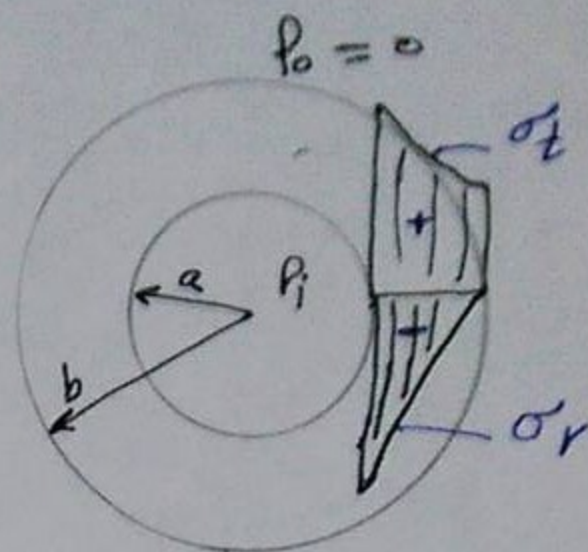
$$\sigma_r = 0$$

$$A = P_i \frac{a^2}{(b^2 - a^2)}$$

$$B = P_i \frac{a^2 b^2}{(b^2 - a^2)}$$

$$\sigma_r = \frac{P_i a^2}{b^2 - a^2} \left(1 - \frac{b^2}{r^2}\right) \rightarrow (10)$$

$$\sigma_t = \frac{P_i a^2}{b^2 - a^2} \left(1 + \frac{b^2}{r^2}\right) \rightarrow (11)$$



Case (b) External pressure only.

$$\sigma_r = 0 \quad \text{at } r=a$$

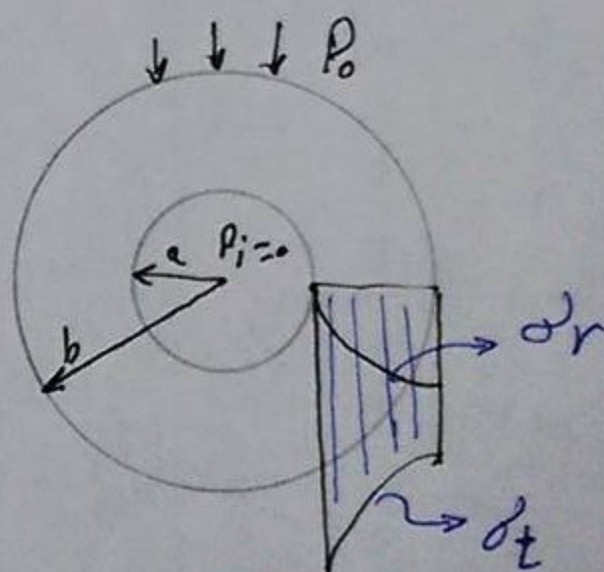
$$\sigma_r = -P_o \quad \text{at } r=b$$

$$A = P_o \frac{b^2}{b^2 - a^2}$$

$$B = P_o \frac{a^2 b^2}{b^2 - a^2}$$

$$\sigma_r = - \frac{P_o b^2}{(b^2 - a^2)} \left(1 - \frac{a^2}{r^2}\right) \rightarrow (12)$$

$$\sigma_t = - \frac{P_o b^2}{(b^2 - a^2)} \left(1 + \frac{a^2}{r^2}\right) \rightarrow (13)$$

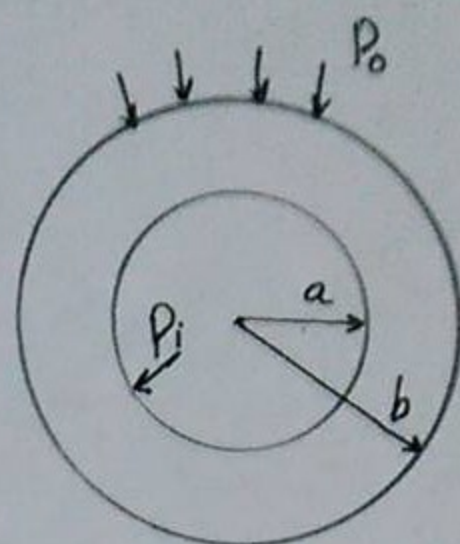


Case 3 Internal pressure & external pressure.

$$\sigma_r = -P_i \rightarrow \text{at } r=a$$

$$\sigma_r = -P_o \rightarrow \text{at } r=b$$

$$\sigma_r = \frac{a^2 P_i - b^2 P_o}{(b^2 - a^2)} - \frac{(P_i - P_o) a^2 b^2}{r^2 (b^2 - a^2)}$$



$$\sigma_t = \frac{a^2 P_i - b^2 P_o}{b^2 - a^2} + \frac{(P_i - P_o) a^2 b^2}{r^2 (b^2 - a^2)}$$

Deformation of the cylinders.

$$\epsilon_r = \frac{1}{E} (\sigma_r - \nu \sigma_t) \rightarrow -\frac{1}{E} \left[A - \frac{B}{r^2} - \nu A - \nu \frac{B}{r^2} \right] = \frac{1}{E} \left[A(1-\nu) - \frac{B}{r^2} (1+\nu) \right] \rightarrow (16)$$

$$\epsilon_t = \frac{1}{E} (\sigma_t - \nu \sigma_r)$$

$$\epsilon_t = \frac{1}{E} \left[A(1+\nu) + \frac{B}{r^2} (1+\nu) \right] \rightarrow (17)$$

$$\therefore \epsilon_t = \frac{u}{r} \quad u = r \cdot \epsilon_t$$

$$\therefore u = \frac{1}{E} \left[A(1+\nu)r + \frac{B}{r} (1+\nu) \right] \rightarrow (18)$$

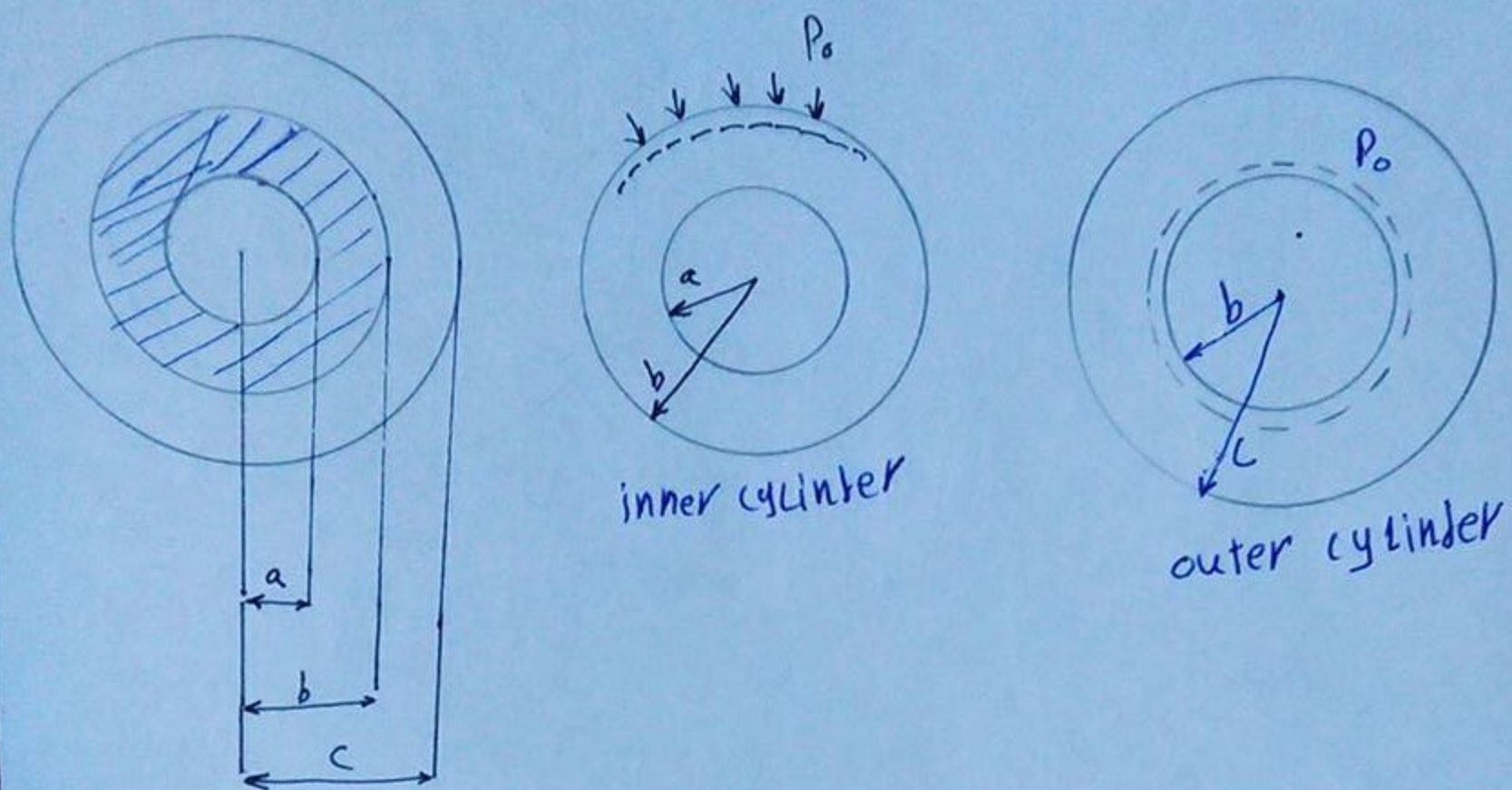
$$\epsilon_z = \frac{1}{E} (\sigma_z - \nu \sigma_r - \nu \sigma_t)$$

$$\text{For long pipe } \epsilon_z = 0$$

$$\text{short pipe } \sigma_z = 0$$

Pressures due to shrink Fits

E



$$P = \frac{E \cdot \delta}{b} \frac{(c^2 - b^2)(b^2 - a^2)}{2b^2(c^2 - a^2)} \rightarrow (19)$$

$$(\sigma_r)_{r=b} = -P \rightarrow (20)$$

$$(\sigma_t)_{r=b} = + \frac{P(b^2 + c^2)}{c^2 - b^2} \rightarrow (21)$$

$$\epsilon_{max} = \frac{1}{2} (\sigma_t - \sigma_r) = \frac{Pc^2}{c^2 - b^2}$$